

Integration by Parts

المجموعة A تمارين مقالية

في التمارين (1-14)، أوجد التكامل.

(1)

$$\int x \cos(3x) dx$$

$$u = x, \quad dv = \cos(3x) dx$$

$$du = dx, \quad v = \frac{1}{3} \sin(3x)$$

$$\int u dv = uv - \int v du$$

$$\begin{aligned} \int x \cos(3x) dx &= \frac{1}{3} x \sin(3x) - \int \frac{1}{3} \sin(3x) dx \\ &= \frac{1}{3} x \sin(3x) + \frac{1}{9} \cos(3x) + C \end{aligned}$$

(2)

$$\int x \sin(5x) dx$$

$$u = x, \quad dv = \sin(5x) dx$$

$$du = dx, \quad v = -\frac{1}{5} \cos(5x)$$

$$\int u dv = uv - \int v du$$

$$\begin{aligned}\int x \sin(5x) dx &= -\frac{1}{5}x \cos(5x) + \int \frac{1}{5} \cos(5x) dx \\ &= -\frac{1}{5}x \cos(5x) + \frac{1}{25} \sin(5x) + C\end{aligned}$$

(3)

$$\int x e^{x-3} dx$$

$$u = x, \quad dv = e^{x-3} dx$$

$$du = dx, \quad v = e^{x-3}$$

$$\int u dv = uv - \int v du$$

$$\begin{aligned}\int x e^{x-3} dx &= x e^{x-3} - \int e^{x-3} dx \\ &= x e^{x-3} - e^{x-3} + C = (x-1)e^{x-3} + C\end{aligned}$$

(4)

$$\int (x-5)e^{x-5} dx$$

$$u = x-5, \quad dv = e^{x-5} dx$$

$$du = dx, \quad v = e^{x-5}$$

$$\int u dv = uv - \int v du$$

$$\begin{aligned}\int (x-5)e^{x-5} dx &= (x-5)e^{x-5} - \int e^{x-5} dx \\ &= (x-5)e^{x-5} - e^{x-5} + C = (x-6)e^{x-5} + C\end{aligned}$$

(5)

$$\int \ln \sqrt[4]{x} dx$$

$$u = \ln \sqrt[4]{x} = \frac{1}{4} \ln x, \quad dv = dx$$

$$du = \frac{1}{4x} dx, \quad v = x$$

$$\int u dv = uv - \int v du$$

$$\int \ln \sqrt[4]{x} dx = x \ln \sqrt[4]{x} - \int \frac{1}{4} dx = x \ln \sqrt[4]{x} - \frac{1}{4} x + C$$

(6)

$$\int \ln(2x - 1) dx$$

$$u = \ln(2x - 1), \quad dv = dx$$

$$du = \frac{2}{2x - 1} dx, \quad v = x$$

$$\int u dv = uv - \int v du$$

$$\int \ln(2x - 1) dx = x \ln(2x - 1) - \int \frac{2x}{2x - 1} dx$$

$$= x \ln(2x - 1) - \int \frac{2x - 1 + 1}{2x - 1} dx$$

$$= x \ln(2x - 1) - \int \left(1 + \frac{1}{2x - 1}\right) dx$$

$$= x \ln(2x - 1) - x - \frac{1}{2} \ln|2x - 1| + C$$

www.kwedufiles.com

$$= x \ln(2x - 1) - x - \frac{1}{2} \ln(2x - 1) + C$$

(7)

$$\int (2x + 1) \ln(x + 1) dx$$

$$u = \ln(x + 1), \quad dv = (2x + 1) dx$$

$$du = \frac{1}{x + 1} dx, \quad v = x^2 + x$$

$$\int u dv = uv - \int v du$$

$$\int (2x + 1) \ln(x + 1) dx = (x^2 + x) \ln(x + 1) - \int \frac{x^2 + x}{x + 1} dx$$

$$= (x^2 + x) \ln(x + 1) - \int \frac{x(x + 1)}{x + 1} dx$$

$$= (x^2 + x) \ln(x + 1) - \int x dx$$

$$= (x^2 + x) \ln(x + 1) - \frac{x^2}{2} + C$$

(8)

$$\int \frac{\ln x}{x^2} dx$$

$$u = \ln x, \quad dv = \frac{1}{x^2} dx = x^{-2} dx$$

$$du = \frac{1}{x} dx, \quad v = -\frac{1}{x}$$

$$\int u dv = uv - \int v du$$

$$\int \frac{\ln x}{x^2} dx = -\frac{1}{x} \ln x + \int x^{-2} dx = -\frac{1}{x} \ln x - \frac{1}{x} + C$$

(9)

$$\int \frac{\ln x}{\sqrt[3]{x}} dx$$

$$u = \ln x, \quad dv = \frac{1}{\sqrt[3]{x}} dx = x^{-\frac{1}{3}} dx$$

$$du = \frac{1}{x} dx, \quad v = \frac{3}{2} x^{\frac{2}{3}}$$

$$\int u dv = uv - \int v du$$

$$\int \frac{\ln x}{\sqrt[3]{x}} dx = \frac{3}{2} x^{\frac{2}{3}} \ln x - \int \frac{3}{2} x^{-\frac{1}{3}} dx = \frac{3}{2} x^{\frac{2}{3}} \ln x - \frac{9}{4} x^{\frac{2}{3}} + C$$

www.kwedufiles.com

(10)

$$\int x^2 \ln x^2 dx$$

$$u = \ln x^2 = 2 \ln x, \quad dv = x^2 dx$$

$$du = \frac{2}{x} dx, \quad v = \frac{x^3}{3}$$

$$\int u dv = uv - \int v du$$

$$\int x^2 \ln x^2 dx = \frac{x^3}{3} \ln x^2 - \int \frac{2}{3} x^2 dx = \frac{x^3}{3} \ln x^2 - \frac{2}{9} x^3 + C$$

(11)

$$\int (x^2 - 2x) \cos x \, dx$$

$$u = x^2 - 2x, \quad dv = \cos x \, dx$$

$$du = (2x - 2)dx, \quad v = \sin x$$

$$\int u \, dv = uv - \int v \, du$$

$$\int (x^2 - 2x) \cos x \, dx = (x^2 - 2x) \sin x - \int (2x - 2) \sin x \, dx \quad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int (2x - 2) \sin x \, dx$$

$$u = 2x - 2, \quad dv = \sin x \, dx$$

$$du = 2dx, \quad v = -\cos x$$

$$\int (2x - 2) \sin x \, dx = -(2x - 2) \cos x + \int 2 \cos x \, dx$$

$$= -(2x - 2) \cos x + 2 \sin x + C \quad (2)$$

من (1) , (2) نحصل على:

$$\int (x^2 - 2x) \cos x \, dx = (x^2 - 2x) \sin x + (2x - 2) \cos x - 2 \sin x + C$$

$$= (x^2 - 2x - 2) \sin x + (2x - 2) \cos x + C$$

(12)

$$\int (x^2 + 3x) \sin x \, dx$$

$$u = x^2 + 3x, \quad dv = \sin x \, dx$$

$$du = (2x + 3)dx, \quad v = -\cos x$$

$$\int u dv = uv - \int v du$$

$$\int (x^2 + 3x) \sin x dx = -(x^2 + 3x) \cos x + \int (2x + 3) \cos x dx \quad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int (2x + 3) \cos x dx$$

$$u = 2x + 3, \quad dv = \cos x dx$$

$$du = 2dx, \quad v = \sin x$$

$$\int (2x + 3) \cos x dx = (2x + 3) \sin x - \int 2 \sin x dx$$

$$= (2x + 3) \sin x + 2 \cos x + C \quad (2)$$

من (1) , (2) نحصل على:

www.kwedufiles.com

$$\int (x^2 + 3x) \sin x dx = -(x^2 + 3x) \cos x + (2x + 3) \sin x + 2 \cos x + C$$

$$= (-x^2 - 3x + 2) \cos x + (2x + 3) \sin x + C$$

(13)

$$\int x^2 e^{x+1} dx$$

$$u = x^2, \quad dv = e^{x+1} dx$$

$$du = 2x dx, \quad v = e^{x+1}$$

$$\int u dv = uv - \int v du$$

$$\int x^2 e^{x+1} dx = x^2 e^{x+1} - \int 2x e^{x+1} dx \quad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int 2xe^{x+1} dx$$

$$u = 2x, \quad dv = e^{x+1} dx$$

$$du = 2dx, \quad v = e^{x+1}$$

$$\int 2xe^{x+1} dx = 2xe^{x+1} - \int 2e^{x+1} dx = 2xe^{x+1} - 2e^{x+1} + C \quad (2)$$

من (2) , (1) نحصل على:

$$\begin{aligned} \int x^2 e^{x+1} dx &= x^2 e^{x+1} - 2xe^{x+1} + 2e^{x+1} + C \\ &= (x^2 - 2x + 2)e^{x+1} + C \end{aligned}$$

(14)

$$\int x^2 e^{2x-3} dx$$

$$u = x^2, \quad dv = e^{2x-3} dx$$

$$du = 2x dx, \quad v = \frac{1}{2} e^{2x-3}$$

$$\int u dv = uv - \int v du$$

$$\int x^2 e^{2x-3} dx = \frac{1}{2} x^2 e^{2x-3} - \int x e^{2x-3} dx \quad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int x e^{2x-3} dx$$

$$u = x, \quad dv = e^{2x-3} dx$$

$$du = dx, \quad v = \frac{1}{2} e^{2x-3}$$

$$\int x e^{2x-3} dx = \frac{1}{2} x e^{2x-3} - \int \frac{1}{2} e^{2x-3} dx = \frac{1}{2} x e^{2x-3} - \frac{1}{4} e^{2x-3} + C \quad (2)$$

من (1) , (2) نحصل على:

$$\begin{aligned} \int x^2 e^{2x-3} dx &= \frac{1}{2} x^2 e^{2x-3} - \frac{1}{2} x e^{2x-3} + \frac{1}{4} e^{2x-3} + C \\ &= \left(\frac{1}{2} x^2 - \frac{1}{2} x + \frac{1}{4} \right) e^{2x-3} + C \end{aligned}$$

(15)

$$\int (\ln x)^2 dx$$

$$u = (\ln x)^2, \quad dv = dx$$

$$du = 2(\ln x) \frac{dx}{x}, \quad v = x$$

$$\int u dv = uv - \int v du$$

$$\int (\ln x)^2 dx = x (\ln x)^2 - \int 2 \ln x dx \quad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int 2 \ln x dx$$

$$u = 2 \ln x, \quad dv = dx$$

$$du = \frac{2}{x} dx, \quad v = x$$

$$\int 2 \ln x dx = 2x \ln x - \int 2 dx = 2x \ln x - 2x + C \quad (2)$$

من (1) , (2) نحصل على:

$$\int (\ln x)^2 dx = x (\ln x)^2 - 2x \ln x + 2x + C$$

(16)

$$\int e^{2x} \sin x \, dx$$

$$u = e^{2x},$$

$$dv = \sin x \, dx$$

$$du = 2e^{2x} dx,$$

$$v = -\cos x$$

$$\int u \, dv = uv - \int v \, du$$

$$\int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2 \int e^{2x} \cos x \, dx \quad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int e^{2x} \cos x \, dx$$

$$u = e^{2x},$$

$$dv = \cos x \, dx$$

$$du = 2e^{2x} dx,$$

$$v = \sin x$$

$$\int e^{2x} \cos x \, dx = e^{2x} \sin x - 2 \int e^{2x} \sin x \, dx \quad (2)$$

نعوض (2) في (1) :

$$\int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2 \left(e^{2x} \sin x - 2 \int e^{2x} \sin x \, dx \right)$$

$$\int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2e^{2x} \sin x - 4 \int e^{2x} \sin x \, dx$$

$$5 \int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2e^{2x} \sin x$$

$$\therefore \int e^{2x} \sin x \, dx = \frac{1}{5} (-e^{2x} \cos x + 2e^{2x} \sin x)$$

(17)

$$\int \sin(\ln x) dx$$

$$u = \sin(\ln x),$$

$$dv = dx$$

$$du = \frac{\cos(\ln x)}{x} dx,$$

$$v = x$$

$$\int u dv = uv - \int v du$$

$$\int \sin(\ln x) dx = x \sin(\ln x) - \int \cos(\ln x) dx \quad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int \cos(\ln x) dx$$

$$u = \cos(\ln x),$$

$$dv = dx$$

$$du = \frac{-\sin(\ln x)}{x} dx,$$

$$v = x$$

$$\int \cos(\ln x) dx = x \cos(\ln x) + \int \sin(\ln x) dx \quad (2)$$

نعوض (2) في (1) :

$$\int \sin(\ln x) dx = x \sin(\ln x) - x \cos(\ln x) - \int \sin(\ln x) dx$$

$$2 \int \sin(\ln x) dx = x \sin(\ln x) - x \cos(\ln x) + C$$

$$\therefore \int \sin(\ln x) dx = \frac{1}{2} x (\sin(\ln x) - \cos(\ln x)) + C$$

المجموعة B تمارين موضوعية

1. a
2. a
3. a
4. b
5. a
6. b
7. b
8. d
9. b
10. b
11. c

www.kwedufiles.com